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R.K. Cohn et al. (MI S U) "The Accuracy of Remapping Irregularly Spaced Velocity Data onto a Regular  
Grid and the Computation of Vorticity"

(Statement A)

## The Accuracy of Remapping Irregularly Spaced Velocity Data onto a Regular Grid and the Computation of Vorticity

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The velocity data obtained from Molecular Tagging Velocimetry (MTV) are typically located on an irregularly spaced measurement grid. In this method of velocimetry, the flowing medium is premixed with a molecular complex that can be turned into a long life-time tracer upon excitation by photons. The velocity vector is determined from the displacement of small regions "tagged" by a pulsed laser which are imaged at two successive times within the lifetime of the tracer. This technique may be viewed as the *molecular* counterpart of PIV. To take advantage of standard data processing techniques, the MTV data need to be remapped onto a regular grid with a uniform spacing. In this work we examine the accuracy and noise issues related to the use of various low order polynomial least-square fits for remapping and the subsequent computation of vorticity from these data. The information obtained has relevance to PIV data processing as well. As noted by Spedding and Rignot (1993), the best estimate of the location of the velocity vector acquired through the use of tracer techniques, such as PIV, is at the midpoint of the displacement vector. Thus, unless special care is taken, PIV data are also initially obtained on an irregular grid.

In the past, various methods have been used for remapping randomly spaced velocity data onto a regular grid. Among them are an inverse distance approach and a "global basis function" examined by Spedding and Rignot (1993). In this study, we consider the use of 2<sup>nd</sup>, 3<sup>rd</sup>, or 4<sup>th</sup> order polynomial to remap the velocity field by performing a local least-squares fit of the irregularly spaced data within region of radius  $R$ . This choice was selected based on the work of Agui and Jimenez (1987), who report that low order polynomial fits and "kriging" techniques produce the most accurate representation of the actual velocity field. However, no quantitative information on the performance of these methods is presented. Four approaches are assessed here for computing the out-of-plane vorticity field from the in-plane velocity measurements: direct differentiation of the polynomial fits used in the remapping process, 1<sup>st</sup> and 2<sup>nd</sup> order finite difference techniques (2<sup>nd</sup> and 4<sup>th</sup> order accurate, respectively) and an 8-point circulation method on the regular data. It should be noted that the 8-point circulation method is identical to the 1<sup>st</sup> order finite difference method after an appropriate choice of smoothing.

Several authors have examined the accuracy of the various means to compute vorticity from regularly spaced data. Spedding and Rignot (1993) used a 1<sup>st</sup> order finite difference technique for the inverse distance method or directly differentiating the global basis function. Results indicated that the global basis function produced generally superior results, however, results were found to be highly dependent upon the ratio, of a characteristic length scale, of the flow,  $L$ , to the mean spacing between measurements,  $\delta$ . Abrahamson and Lonnes (1995) found that the circulation method resulted in slightly more accurate vorticity results than using a least-squares fit to a model velocity field. Luff *et al.* (1999) compared the 1<sup>st</sup> and 2<sup>nd</sup> order finite difference methods and an 8-point circulation method in the calculation of vorticity in the presence of both noise and missing data points. In terms of only the computed vorticity RMS, the 1<sup>st</sup> order finite difference technique produced the best results.

One short-coming of the above mentioned studies is that only the random component of the error field is examined. Fouras and Soria (1998) found that the error in the vorticity field could be

better represented if it is divided into two portions: a mean bias error due to spatial filtering, and a random error resulting from the propagation of error in the velocity measurements into the calculation of vorticity. In some cases, the mean bias error can be significantly larger than the random error. The Fouras and Soria study found that differentiating a 2<sup>nd</sup> order polynomial least-squares fit to the velocity data produced results superior to the 1<sup>st</sup> order finite difference approach. However, the results based on differentiating the fit were sensitive to the number of points used in the fit. This work was based entirely on regularly sampled velocity data; issues connected to remapping an irregular data set were not considered.

The aforementioned investigations suggest different optimum methods for vorticity computation depending on the criterion used to assess the error. In our work we directly compare several of the different vorticity calculation methods which were determined in the previous studies to produce the best results. In addition, we include the effect of remapping of the velocity field on the accuracy of vorticity estimation. The differentiation of the polynomial fit, used in remapping, as a means of estimating the vorticity is also considered. A simulation of an Oseen vortex, utilized by the previous studies, is also employed here as the basis for comparison. The effect of uncertainty in the velocity measurements is simulated by adding a random amount of noise to each velocity component.

Our simulations show that the accuracy of the remapping process depends on both  $L/\delta$  and the normalized size of the region used in the least-squares fit,  $R/\delta$ . The effect of increasing the value of  $R/\delta$  on the velocity and vorticity fields is to decrease the random error, but increase the mean bias error. Generally, the decrease in random error is smaller than the increase in the bias error. All of the different polynomial orders tested produced accurate results in the remapping of the velocity field for suitable choice of  $L/\delta$  and  $R/\delta$ . For example, with  $L/\delta > 4.5$  both the velocity mean bias error and random error are less than 1% of the peak velocity.

Results show that the most accurate vorticity results are achieved by either directly differentiating the 3<sup>rd</sup> order polynomial fit to the original irregular data or by the 2<sup>nd</sup> order finite difference technique. Note that this conclusion is different from previous studies because they either did not consider the 2<sup>nd</sup> order finite difference approach, or the performance was based only on the random error. For  $L/\delta > 4.5$ , we find the mean bias error and random error in vorticity can be less than 3% and 2%, respectively, of the peak vorticity. A decrease in  $L/\delta$  causes a large increase in the vorticity bias error; error values higher than 18% of the peak vorticity are found for  $L/\delta = 2.5$ .

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